Zoom Class 30/04/2020

Ques Find the asymptotes of the curve
$$x^3 - 2y^3 + 2x^2y - xy^2 + xy - y^2 + 1 = 0$$

Soln Given, $x^3 - 2y^3 + 2x^2y - x y^2 + xy - y^2 + 1 = 0$

$$\Rightarrow x^3\left\{1 - 2\left(\frac{y}{x}\right)^3 + 2\left(\frac{y}{x}\right) - \left(\frac{y}{x}\right)^2 + \left(\frac{y}{x}\right)x - \left(\frac{y}{x}\right)^2 + \frac{1}{x^3}\right\} = 0$$

$\therefore$ Highest degree is $n = 3$

$\therefore$ Putting $x = 1, y = m$ in third degree terms we get

$$\phi_3(m) \quad 1 - 2m^3 + 2m - m^2 = 0$$
$$\Rightarrow (1+2m) - m^2(1+2m) = 0$$
$$\Rightarrow (1+2m)(1-m^2) = 0$$
$$\Rightarrow m = \left(\frac{-1}{2}\right), \pm 1 \Rightarrow m = 1, -1, (-1/2)$$

Again $\phi_2(m) = m - m^2 = 0$
$$\Rightarrow m(1-m) = 0 \Rightarrow \phi_2(m) = m(1-m)$$

and $\phi_3'(m) = 0 - 6m^2 + 2 - 2m = -6m^2 - 2m + 2$

$$\Rightarrow c = -\frac{\phi_2(m)}{\phi_3'(m)} = \frac{m(1-m)}{6m^2+2m-2}$$

$\therefore$ for $m = 1 \Rightarrow c = \frac{1(0)}{6+2-2} = \frac{0}{6} = 0$

for $m = -1 \Rightarrow c = \frac{-1(1+1)}{6-2-2} = \frac{-2}{6-4} = \frac{-2}{2} = -1$

for $m = -\frac{1}{2}$

$$\Rightarrow c = \frac{\left(-\frac{1}{2}\right)\left(1+\frac{1}{2}\right)}{\cancel{6}3\left(\frac{1}{\cancel{4}2}\right) - \frac{\cancel{2}}{2} - 2}$$

$$= \frac{\left(-\frac{1}{2}\right)\left(\frac{3}{2}\right)}{\frac{3}{2} - 3} = \frac{-3}{\cancel{4}2} \times \frac{\cancel{2}}{(3-6)}$$

$$= \frac{-\cancel{3}}{2} \times \frac{1}{-\cancel{3}} = \frac{1}{2}$$

$\therefore$ The required asymptotes are

$$y = 1\cdot x + 0 \Rightarrow \boxed{y = x}$$
$$y = -1\cdot x - 1 \Rightarrow \boxed{y + x + 1 = 0}$$
$$y = -\frac{1}{2}x + \frac{1}{2} \Rightarrow \boxed{2y + x - 1 = 0}$$

Ans

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Ques Find the asymptotes of
$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$$

Soln Given equation is
$$4x^3 - 3xy^2 - y^3 + 2x^2 - xy - y^2 - 1 = 0$$
the highest degree of eqn is 3

$\therefore$ Putting $x = 1$ and $y = m$ in 3rd degree terms we get

$$\phi_3(m) = 4 - 3m^2 - m^3 = 0$$
$$\Rightarrow m^3 + 3m^2 - 4 = 0 \Rightarrow (m-1)(m^2 + 4m + 4) = 0$$
$$\Rightarrow (m-1)(m+2)^2 = 0$$

$\Rightarrow m = 1, -2, -2$

Again $\phi_2(m) = 2 - m - m^2$

$= -(m^2 + m - 2)$

$= -(m^2 + 2m - m - 2)$

$= -(m-1)(m+2)$

and $\phi_3'(m) = 0 - 6m - 3m^2$

$= -(6m + 3m^2)$

Now $c = -\dfrac{\phi_2(m)}{\phi_3'(m)} = -\left[\dfrac{\cancel{-}(m-1)(m+2)}{\cancel{-}(6m+3m^2)}\right]$

$= -\dfrac{(m-1)\cancel{(m+2)}}{3m\cancel{(m+2)}} = -\dfrac{(m-1)}{3m}$

For $m = 1$. $c = -\dfrac{(1-1)}{3} = 0$

for $m = -2$ $\Rightarrow c = -\dfrac{(-2-1)}{3(-2)} = \dfrac{\cancel{-}3^{1}}{\cancel{-}6_{2}} = \dfrac{1}{2}$

But $c = -\dfrac{(m-1)}{3m}$ only when $(m+2) \neq 0$

and for $m = -2$, $m+2 = 0$

$\therefore c = \dfrac{0}{0}$

So we use the formula

$\dfrac{c^2}{2!}\phi_3''(m) + c\,\phi_2'(m) + \phi_1(m) = 0$

$\Rightarrow \dfrac{c^2}{2!}(-6-6m) + c(-1-2m) + (0) = 0$

$$\Rightarrow -3c^2(1+m) - c(1+2m) = 0$$
$$\Rightarrow 3c^2(1+m) + c(1+2m) = 0$$
$$\Rightarrow c\{3c(1+m) + (1+2m)\} = 0$$

$$\Rightarrow c = 0 \text{ or } \{3c(1+m) + (1+2m)\} = 0$$
$$\Rightarrow c = 0 \text{ or } \{3c(1-2) + (1-4)\} = 0 \quad \{\because m = -2\}$$
$$\Rightarrow c = 0 \text{ or } \{-3c - 3 = 0\}$$
$$\Rightarrow c = 0 \text{ or } c = -1$$

$\therefore$ Required ~~equation of~~ Asymptotes are

$$y = 1\cdot x + 0 \Rightarrow \boxed{y = x}$$
$$y = -2\cdot x + 0 \Rightarrow \boxed{y - 2x = 0}$$
$$y = -2\cdot x + (-1) \Rightarrow \boxed{y + 2x + 1 = 0}$$

Working Rule for asymptotes parallel to axes

If the asymptote is parallel to x-axis then we put co-efficicients of x^n and x^{n-1} equal to zero where 'n' is degree of the equation.
If x^n is absent then co-efficient of x^{n-1} to zero.
If both x^n and x^{n-1} are absent, then the equation of asymptotes is found by equating x^{n-2} equal to zero.

Similarly to find asymptote is parallel to y-axis we equate y^n and y^{n-1} to zero. If y^n is not present then y^{n-1} is equated to zero.

If both y^n and y^{n-1} are not present then y^{n-2} is equated to zero to find the equation of asymptotes.

Ques Find all the asymptotes of the curve $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

Soln Given $\frac{a^2}{x^2} + \frac{b^2}{y^2} = 1$

$$\Rightarrow a^2y^2 + b^2x^2 = x^2y^2 \quad \text{——— (1)}$$

Clearly the equation of curve is of 4^{th} degree. but x^4, x^3, y^4, y^3 are absent so taking co-efficient of x^2 we get

$$\Rightarrow a^2y^2 = x^2y^2 - b^2x^2$$

$$\Rightarrow a^2y^2 = (y^2 - b^2)x^2$$

So $y^2 - b^2 = 0 \Rightarrow y^2 = b^2 \Rightarrow y = \pm b$

Similarly for y^2

$$a^2y^2 - x^2y^2 = -b^2x^2$$

$$\Rightarrow (a^2 - x^2)y^2 + b^2x^2 = 0$$

So $a^2 - x^2 = 0 \Rightarrow a^2 = x^2 \Rightarrow x = \pm a$

So the required equation of asymptotes are

$$x = \pm a, \; y = \pm b$$

Ans